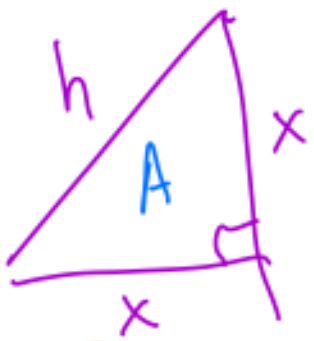
Example: If $f(x) = \arccos(x)$, what is $f'(x)$, $f''(x)$?

$$f'(x) = \boxed{\frac{-1}{\sqrt{1-x^2}}} = -(1-x^2)^{-1/2}$$

$$f''(x) = \frac{1}{2}\cdot(1-x^2)^{-3/2}\cdot(-2x) = \boxed{\frac{-x}{(1-x^2)^{3/2}}}$$

4.45 How fast is the area of an isosceles right triangle growing when the hypotenuse is 6 inches long and is growing at a rate of 1 inch per minute?



Given $h = 6$, $\frac{dh}{dt} = 1 \text{ in/min}$

$\frac{dA}{dt} = ?$

$A = \frac{1}{2}x^2 = \frac{h^2}{4}$

Pyth: $x^2 + x^2 = h^2$, $2x^2 = h^2$, $x^2 = h^2/2$

$h = \sqrt{2}x$

$A = \frac{h^2}{4}$

$\frac{dA}{dt} = \frac{1}{4}\left(2h\frac{dh}{dt}\right)$

$\frac{dA}{dt} = \frac{h}{2}\cdot\frac{dh}{dt}$

$\frac{dA}{dt} = \frac{6 \text{ in}}{2}\cdot 1 \text{ in/min}$

$= \boxed{3 \text{ in}^2/\text{min}}$

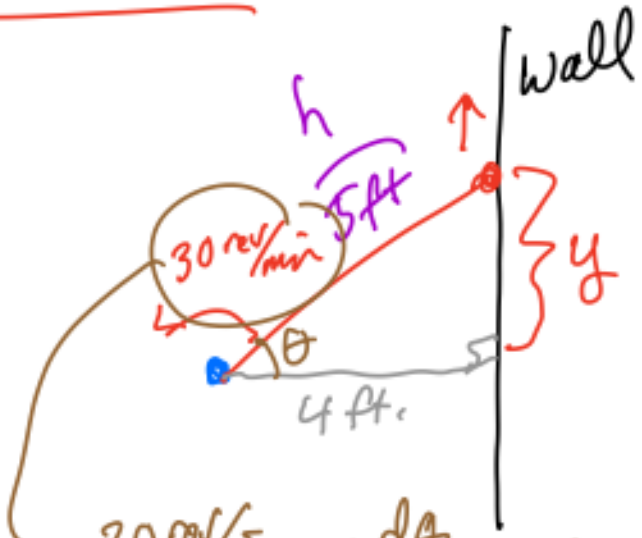
Q: What is $(|x|)'$?

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \Rightarrow (|x|)' = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ \text{undefined} & \text{if } x = 0 \end{cases}$$

4.51 Dr. Richardson wears a laser pointer on the top of his head that rotates horizontally at a rate of 30 revolutions per minute. He stands 4 feet from a dark wall nearby, and

the light makes a moving dot on the wall. When the dot on the wall is exactly 5 feet from Dr. Richardson, how fast is it moving?

Top view



$30\ \text{rev/min} = \frac{d\theta}{dt} = \frac{30\ \text{rev}}{\text{min}} \cdot \frac{2\pi\ \text{radians}}{1\ \text{rev}}$

Need eqn with y & θ

$\tan\theta = \frac{y}{4}$

$\frac{d\theta}{dt} = \frac{60\pi}{\text{min}}$

$\frac{dy}{dt} = ?$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{4}\frac{dy}{dt}$

$\frac{25}{16} \cdot 60\pi = \frac{1}{4}\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{25}{16} \cdot 60\pi \cdot 4$

$= \ldots\ \text{ft/min}$

Example Find the absolute maximum & abs. minimum of the function

$$f(x) = x^4 + x^3 - 12x + 250$$

on the interval

(a) $[-4, 2]$

(b) $[-4, \infty)$

(c) $(-4, 200)$

(a) Cr pts: $f'(x) = 0$

$$4x^3 + 3x^2 - 12 = 0$$

Sagemath:

Type some Sage code below and press Evaluate.

```
1  f(x)=x^4+x^3-12*x+250
2  fp(x)=diff(f(x),x)
3  show(f(x))
4  show(fp(x))
5  ans=solve(fp(x)==0,x)
6  show(ans[0].rhs().n())
7  show(ans[1].rhs().n())
8  show(ans[2].rhs().n())
9  show(ans[2])
```

$\Rightarrow x \approx 1.23$ is the only solution to $f'(x) = 0$ $\Rightarrow$ only critical pt.

Endpoints: $x = -4, 2$

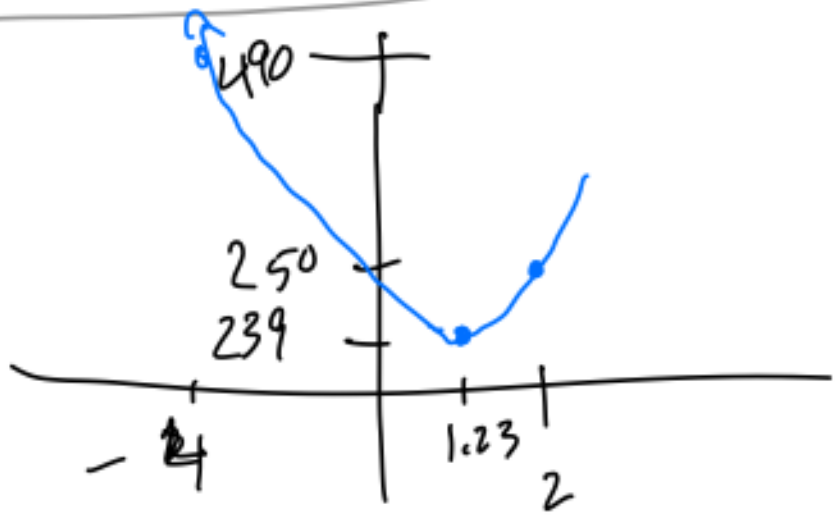
possible abs max & min

	x	$f(x)$
cr pt →	$x \approx 1.23$	239.39
end pts {	$x = -4$	490
	$x = 2$	250

$x = -4$ abs max

abs max value $= 490$

→ $x \approx 1.23$ is abs. min with abs min value 239.39



(b) Interval $[-4, \infty)$

$f(x) = x^4 + x^3 - 12x + 250$

$f'(x) = 4x^3 + 3x^2 - 12 = 0 \Rightarrow x \approx 1.23.$

Investigation: • $f(-4) = 490$

• $f(1.23) = 239.39$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^4+x^3-12x+250$

$= \boxed{\infty}$.

$\Rightarrow$ $x=1.23$ is abs min with abs min value ≈ 239.39

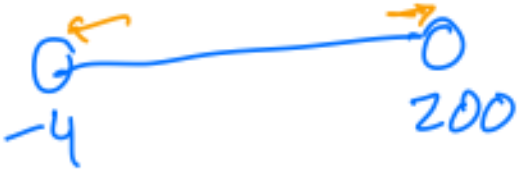
No abs max

(c) $f(x) = x^4+x^3-12x+250$

$[-4, 200)$

$f'(x) = 0 \Rightarrow x = 1.23$ cpt.

investigation $f(1.23) \approx 239.39$



$\lim_{x\to -4^+} f(x) = f(-4) = 490$

$\lim_{x\to 200} f(x) = f(200) = 1607997850$

$\Rightarrow$ abs min: $x=1.23$, value 239.39

abs max: does not exist (would be $x=200$, but not included)

From last time:

Example Let $f(x) = e^{-x^2}$.

Find all critical pts & inflection points, intervals of increase & decrease.

Solution: $f'(x) = -2xe^{-x^2}$

$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$

$= (-2+4x^2)e^{-x^2}$

$f'(x) = 0 \Rightarrow -2x\underbrace{e^{-x^2}}_{\text{always positive}} = 0 \Rightarrow x = 0$ is only cr pt.

$f''(x) = 0 \Rightarrow (-2+4x^2)\underbrace{e^{-x^2}}_{\text{Never zero}} = 0$

$\Rightarrow -2+4x^2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} \approx \pm .7$

$f'(x) = 2xe^{-x^2}$ $+++$ 0 $---$

$f''(x) = +++\ 0\ ---\ 0\ +++$

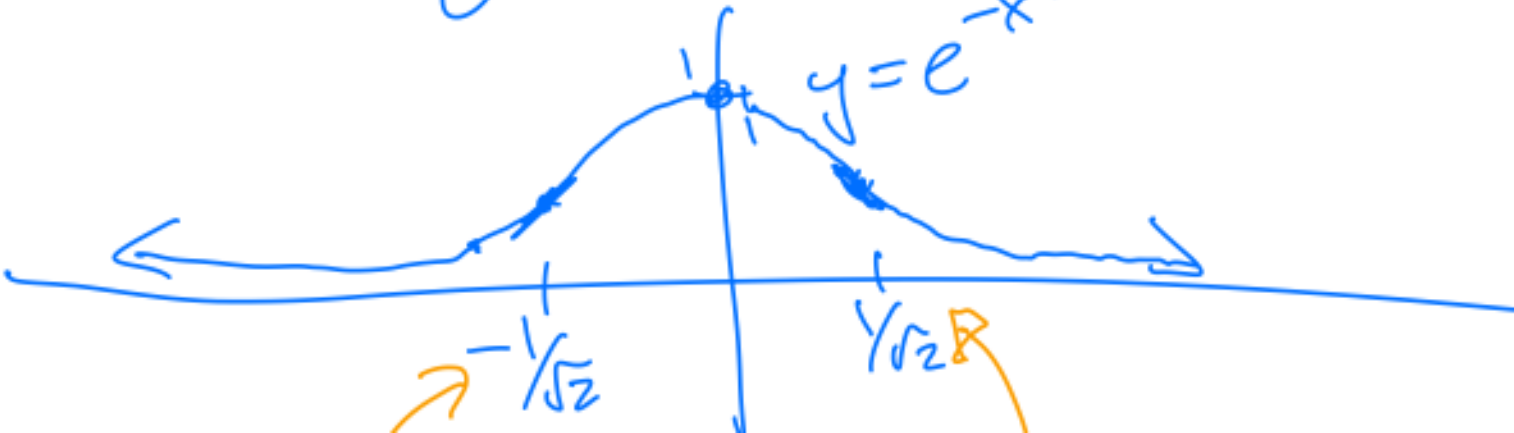
$(-2+4x^2)e^{-x^2}$

$f(x) = e^{-x^2}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} e^{-x^2} = 0 = \lim_{x\to-\infty} e^{-x^2}$$

horiz asymptote at $y = 0$

$e^{-x^2} > 0$



max of $f'(x)$ (slopes)

Min of $f'(x)$

Example Let $f(x) = e^{-x^2}$.
Find all critical pts & inflections, intervals of increase & decrease.

Solution: $f'(x) = -2xe^{-x^2}$

$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$

$= (-2 + 4x^2)e^{-x^2}$

Quiz.

(1) What is your name (including any other initials)?

(2) What score will you try to make on the test Monday?

(3) Simplify $\dfrac{x+1}{x^2-1}$

(4) Find the derivative

(a) $A(x) = x^2 \tan(x)$

(b) $B(t) = \arcsin(2t+3)$